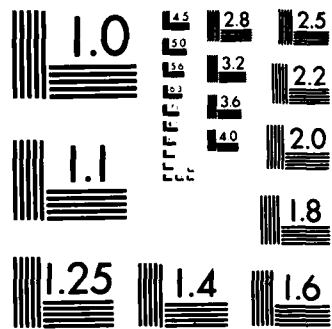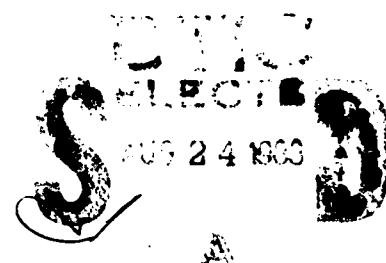END
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A LATENT TRAIT MODEL
FOR DIFFERENTIAL STRATEGIES
IN COGNITIVE PROCESSES

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Some cognitive psychologists, who have tried to approach psychometric theories, say that they do not provide them with theories and methods with which they can deal with differential strategies. They are not exactly right. As early as in the late nineteen-sixties, the heterogeneous case of the graded response level in the context of latent trait theory was proposed (A general model for the operating characteristic of graded item response, Psychometric Laboratory Report No. 55, 1967, University of North Carolina) as a model for cognitive processes. Some useful hints for differential strategies are seen elsewhere (A general model for free-response data, Psychometrika Monograph No. 18, 1972, Section 3.4) under the title, "Multi-correct and multi-incorrect responses."

In the present paper, following the same line, a general latent trait model for differential strategies in cognitive processes is proposed, and the maximum likelihood estimation of the individual's latent trait is discussed.

A LATENT TRAIT MODEL FOR DIFFERENTIAL STRATEGIES

IN COGNITIVE PROCESSES

ABSTRACT

Some cognitive psychologists, who have tried to approach
psychometric theories, say that they do not provide them with theories
and methods with which they can deal with differential strategies.
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the heterogeneous case of the graded response level in the context of
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trait model for differential strategies in cognitive processes is
proposed, and the maximum likelihood estimation of the individual's
latent trait is discussed.

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I Introduction

A general model for free-response data has been proposed by the author (Samejima, 1972) which includes three different response levels, i.e., the nominal response level, the graded response level, and the dichotomous response level. The general graded response model is further classified into the homogeneous and the heterogeneous cases.

Two of the most common uses of the graded response model may be for test items which have more than two item score categories, e.g., A, B, C, D and F, and for questionnaire items with multiple response categories, e.g., "strongly disagree," "disagree," "agree," and "strongly agree." The rationale behind this general model is, however, originally based upon the general sequential cognitive process, which has been described elsewhere (Samejima, 1967).

Let θ be the latent trait underlying the cognitive process. When the latent space is unidimensional, θ is the set of all real numbers, and, when it is multidimensional, θ is a vector of k sets of real numbers, such that

$$(1.1) \qquad \theta = (\theta_1, \theta_2, \cdots, \theta_k)' \quad ,$$

with k indicating the dimensionality. In the present paper, however, we deal solely with the unidimensional latent space.

Let us take problem solving as an example. Suppose that for solving the problem g we need m_g sequential subprocesses. Let y_g denote the attainment category or attainment score. One must

successfully follow all the m_g sequential subprocesses in order to solve the problem g , so the attainment category y_g assumes integers, 0 through (m_g+1) , with $y_g = 0$ indicating that the individual subject has successfully followed no subprocesses, and with $y_g = m_g$ meaning that he has completed all m_g subprocesses required to solve the problem. The additional attainment score, (m_g+1) , represents that the subject has successfully followed the additional subprocess which is hypothesized at the end of the entire sequence of subprocesses. Since no one can accomplish this, the conditional probability, given θ , with which the subject obtained the attainment score y_g equals zero, regardless of the fixed value of θ . With this setting, we can see that the general graded response model can readily be applied to the single strategy case of problem solving. In this paper, however, our main objective is to approach a general model for the multiple strategy case, or differential strategies, in the context of latent trait theory.

The operating characteristic of a discrete response relates the response to the latent trait θ . In the context of problem solving, this operating characteristic is considered with respect to an attainment score, or to a union of attainment scores. It takes an important role in estimating the subject's latent trait from his performance.

As concrete mathematical models, we shall consider those in the homogeneous case, which was expanded and generalized from the one on the graded response level.

II Cognitive Process as Digraphs

It is a fairly common phenomenon that there exist more than one way of solving a problem. In proving mathematical theorems, for example, we often find one proof plus several alternative proofs for one theorem. To represent those differential strategies graphically, Figure 2-1 presents a simple example of a two strategy case in the form of a graph, each strategy having a small number of subprocesses. In this example, if we take the first strategy, then we must traverse the path, $v_0 \ v_1 \ v_2 \ v_3 \ v_4$, whereas we must follow another path $u_0 \ u_1 \ u_2 \ u_3 \ u_4 \ u_5$ if we take the second strategy, in order to solve the problem. Note that, although we have duplicate symbols, $v_0 = u_0$, $v_1 = u_1$, $v_3 = u_4$ and $v_4 = u_5$. These two paths represent the two successful strategies for solving our hypothetical problem, and are shown as two directed subgraphs of the original graph, or digraphs, in Figure 2-2.

When the subject falters, he may not follow either of the two paths shown in Figure 2-2. In such a case, we need additional arcs in the digraph presented as Figure 2-1, examples of which are shown by thin lines in Figure 2-3. Two examples of the directed subgraphs of Figure 2-3, which represent "faltering," are presented in Figure 2-4. In the first digraph, the subject initially took the first strategy, but came back from the vertex v_2 to the vertex v_1 , and then took the second strategy until he solved the problem. In the second digraph, the subject initially took the second strategy up to the vertex u_3 , then came back to the vertex u_1 and took the first strategy. These are rather simple examples adding one cycle to each path in Figure 2-2, making the strategy a trail instead of a
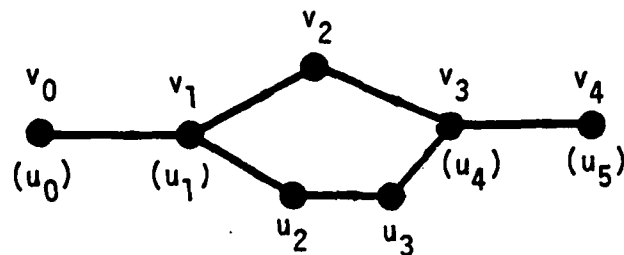
FIGURE 2-1

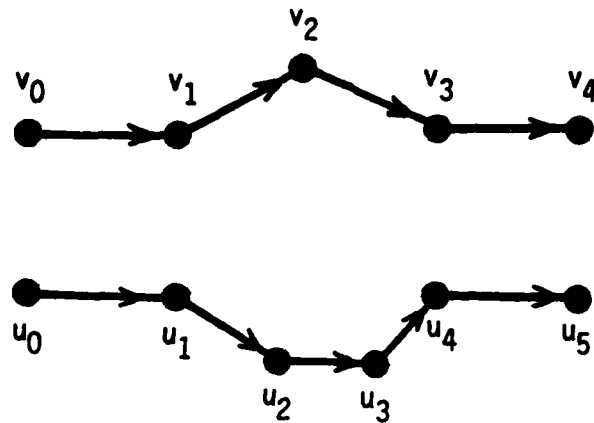Example of Two Strategy Case Drawn as a
Graph.

FIGURE 2-2

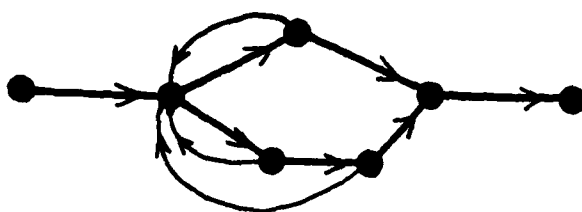Two Directed Subgraphs Representing the
Two Strategies.

FIGURE 2-3

Digraph Including Two Loops and a Cycle
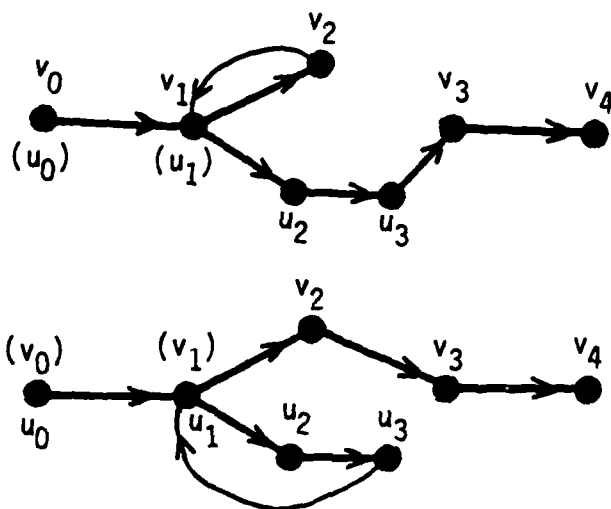Representing the Subject's Faltering.

FIGURE 2-4

Two Examples of Directed Subgraphs
Representing the Subject's
Faltering.

path. We can conceive of more complex examples, however, in which the subject traverses several cycles repeatedly in a single walk, for instance.

So far in our graphs all the vertices are those included in either the first or the second, or both, of the two directed subgraphs of Figure 2-2, which represent the two successful strategies for solving the problem. In our cognitive process, however, we often choose wrong strategies which do not lead to the solution of the problem. Figure 2-5 illustrates such situations, in which hollow circles are added to the solid circles of Figure 2-1, and dashed arcs are added to imply additional paths, which represent wrong strategies. Two examples of such unsuccessful strategies are illustrated in Figure 2-6. In the first subgraph, the subject followed the second successful strategy up to the vertex u_2 , and then branched himself to a subsequence of two subprocesses which led him to a "dead end." The second subgraph illustrates a similar case in which the subject followed the second successful strategy up to the vertex u_3 , and then followed a "dead end" subsequence of two subprocesses.

If the subject took a wrong strategy and found out he could not solve the problem if he sticks to that strategy, he may give up, or he may come back to a previous point in the path and try another strategy. To illustrate this latter situation, additional arcs are necessary in the graph of Figure 2-5. In Figure 2-7, some dotted arcs are added to imply new cycles which the subject may follow when he "comes back." Two examples of such trails are given in Figure 2-8. In the first example, the trail indicates that the subject branched himself to a wrong strategy after he reached the vertex u_2 of the second successful strategy,
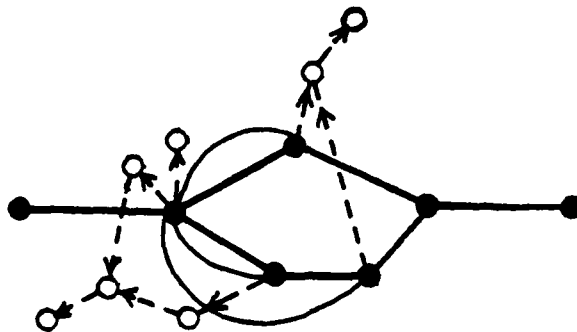
FIGURE 2-5

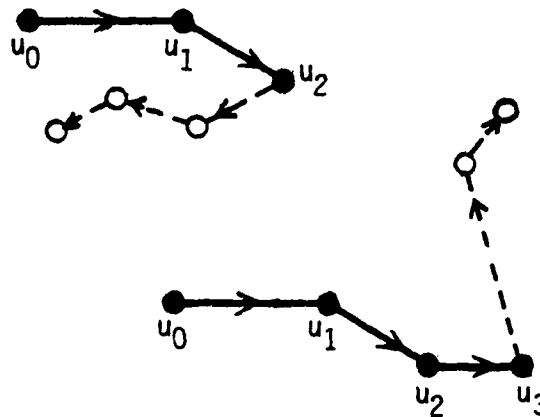Digraph Added by Unsuccessful Strategies
(Dashed Line).

FIGURE 2-6
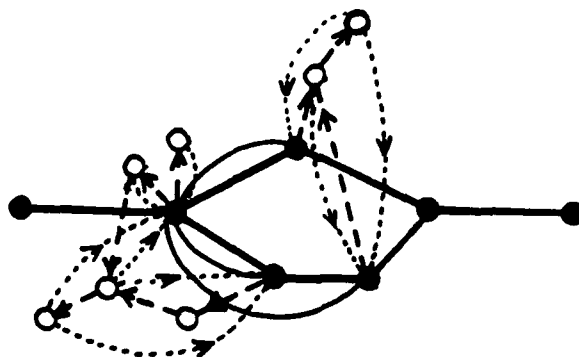
Two Examples of Unsuccessful Strategies.

FIGURE 2-7

Digraph Added by Loops and Cycles Representing
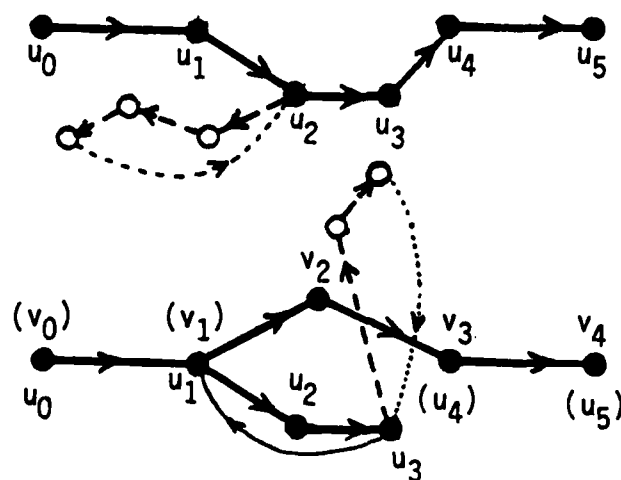the Subject's Faltering after He Has Taken
Unsuccessful Strategies.

FIGURE 2-8

Two Trails Representing the Subject's
Faltering when He Has Taken
Unsuccessful Strategies.

followed the path until he reached the "dead end," and then came back to u_2 and followed the rest of the second successful strategy. In the second example, the subject followed up to the vertex u_3 of the second successful strategy, branched himself to a wrong strategy, followed the path to the "dead end," and came back to u_3 , went back to the vertex u_1 $(= v_1)$, and then took the first successful strategy until he solved the problem.

We have seen just a small number of examples of conceptualizing the cognitive processes involved in problem solving, and there are a great many other varieties of paths, trails, and walks which might represent a particular subject's cognitive process. The subject may walk the same cycles over and over again, for example, or he may stop at, say, the vertex v_2 in the path representing the first successful strategy and then not proceed, and so forth. In any case, graphic representation illustrated in this section will benefit us in systematizing those complex situations.

III Differential Strategy Tree

As was referred to in the first section, latent trait theory relates the subject's behavior to his position in the latent space. In problem solving, his behavior is observed in terms of the degree of attainment toward the solution of the problem.

It is obvious that following those cycles presented in Figures 2-3 and 2-7 itself will not directly improve the subject's degree of attainment toward the solution of the problem. For this reason, as far as

we consider the degree of attainment toward the solution, we can more or less ignore the subject's traversing on cycles. In other words, the things that count are the paths in those graphs, rather than trails or walks which may include one or more cycles. Thus the graph of Figure 2-7 may be reduced to the one presented in Figure 3-1. This implies that, when we consider the subject's degree of attainment toward the solution, the first trails in Figures 2-4 and 2-8 are treated as equivalent to the second path in Figure 2-2, and the second trails in those two figures are treated as equivalent to the first path in Figure 2-2.

We also notice that even the same subject in exactly the same problem solving situation may take different strategies, if it is given repeatedly. Thus even if his latent trait and the given situation stay the same, he may have a higher probability of solving the problem when he chooses one strategy in preference to the others. In other words, there is no reason to assume that for a given individual of trait θ the probability of success stays the same when he chooses different successful strategies. In the illustration presented in the preceding section, the probability with which the subject of trait θ solves the problem using the first successful strategy may not be the same as the one when he uses the second successful strategy, even though the edge $(v_3\ v_4)$ is the same as the edge $(u_4\ u_5)$.

The above observation suggests that, as far as we consider the degree of attainment and its corresponding operating characteristic, it may be more convenient to represent the set of all strategies as a tree. Figure 3-2 presents such a tree obtained from the graph in Figure 3-1 for
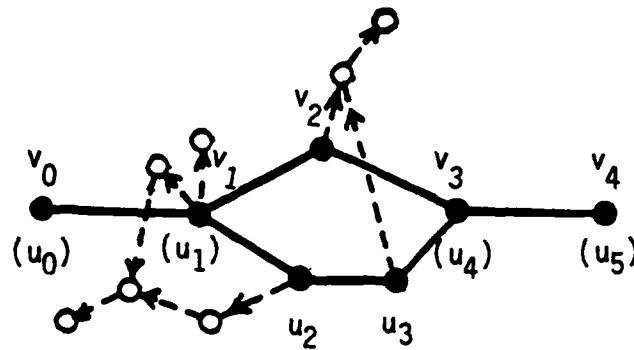
FIGURE 3-1

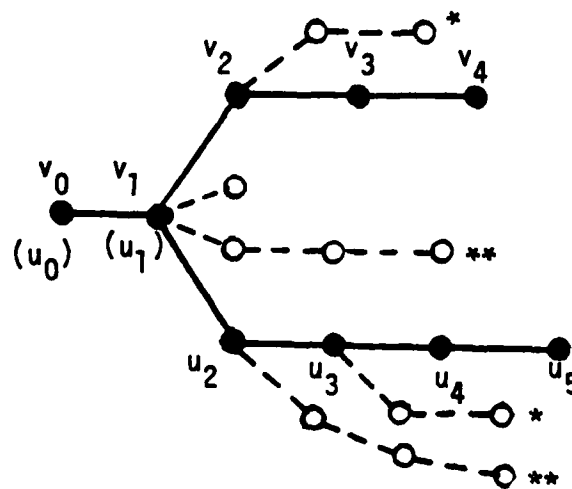Digaraph Excluding All Cycles.

FIGURE 3-2

Differential Strategy Tree Including Both
Successful and Unsuccessful Strategies.

the example discussed in the preceding section. Note that not only are the two points v_4 and u_5 in Figure 3-2 the same point in the original graph, Figure 3-1, but also the two hollow circles marked with * in Figure 3-2 are a single point in Figure 3-1, and so are the two marked with ** . Hereafter, we shall call this kind of tree the differential strategy tree. It is a kind of directed graph which contains several paths representing different strategies, joining a common initial endpoint with the other distinct endpoints. We may call this initial point a nothing point, indicating that, "nothing has been accomplished yet." We may call the other endpoints for the successful strategies solution points, meaning, that the "solution has been reached." Since no one can surpass a solution point, it also represents a hypothesized attainment score which no one can obtain.

In theory, we should be able to estimate the operating characteristic for each of the subprocesses in Figure 3-2, which are represented by solid or dashed edges, if appropriate models are discovered. In practice, however, the development of such a model, or models, and its validation will involve much more complicated problems, if, at the initial stage, we include unsuccessful strategies in addition to the successful strategies. For this reason, here we shall consider models for successful strategies only, and solely use the information related to the successful strategies. When some model thus constructed has been validated, we can proceed to explore the operating characteristics of the various subprocesses of unsuccessful strategies, in a way similar to that of estimating the plausibility function of each distractor of the multiple-choice test item

after we have estimated the operating characteristic of the correct answer (cf. Samejima, ONR Final Report, 1981). Thus the path representing the first unsuccessful strategy in Figure 3-2 is treated here as equivalent to the path $v_0 \ v_1 \ v_2$ in the first successful strategy, those representing the second and third unsuccessful strategies to the path $v_0 \ v_1$ in the first successful strategy, the path for the fourth unsuccessful strategy to the path $u_0 \ u_1 \ u_2 \ u_3$ in the second successful strategy, and the one for the fifth to the path $u_0 \ u_1 \ u_2$ in the second successful strategy.

The example we have observed is simple and easy. In reality, however, we should expect more complicated differential strategy trees. Figures 3-3 and 3-4 present another, a little more complicated example of the digraph and corresponding differential strategy tree, in which only successful strategies are drawn. Thus in our second example, we have five successful strategies and five solution points.

IV A General Model for Differential Strategies

Let w denote the number of successful strategies for solving the problem g . This number equals the number of solution points in the differential strategy tree, which was illustrated in the preceding section. Each of those w strategies consists of m_{gi} $(i = 1,2,...,w)$ subprocesses, and they are represented by the vertices, excluding the first and last, in the digraphs and in the differential strategy trees presented in the previous two sections. In the example which was first presented and discussed in Section 2, $w = 2$, $m_{g1} = 3$ and $m_{g2} = 4$, and the subprocesses are represented by four edges, $(v_0 \ v_1)$, $(v_1 \ v_2)$, $(v_2 \ v_3)$
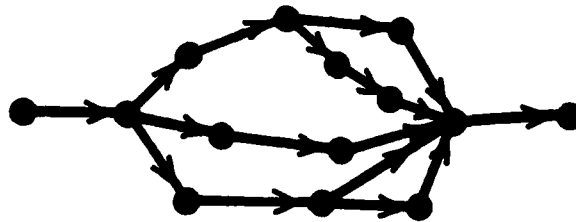
FIGURE 3-3

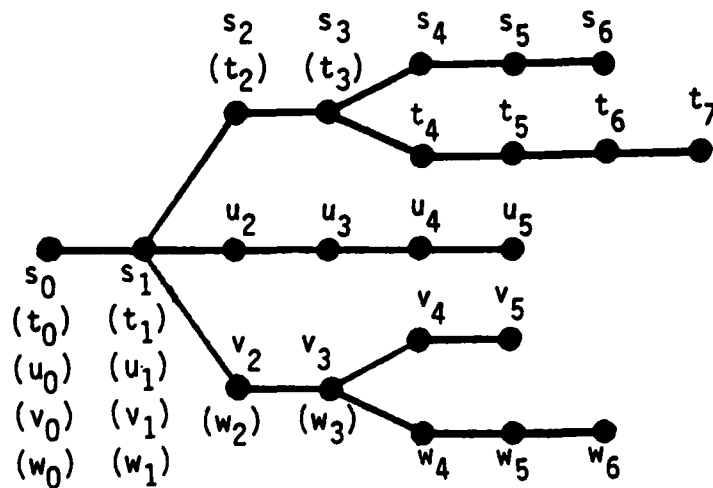Another Example of Digraph Representing
Five Successful Strategies.

FIGURE 3-4

Differential Strategy Tree of the Second
Example Including Successful Strategies
Only.

and $(v_3 \, v_4)$, in the path $v_0 \, v_1 \, v_2 \, v_3 \, v_4$ representing the first successful strategy, and by five edges, $(u_0 \, u_1)$, $(u_1 \, u_2)$, $(u_2 \, u_3)$, $(u_3 \, u_4)$ and $(u_4 \, u_5)$, in the path $u_0 \, u_1 \, u_2 \, u_3 \, u_4 \, u_5$ which represents the second successful strategy. Hereafter, we shall call this example Example 1. It is fairly common that in the original digraph some subprocesses are shared by two or more different strategies, as we can see in Example 1, i.e., $(v_0 \, v_1) = (u_0 \, u_1)$ and $(v_3 \, v_4) = (u_4 \, u_5)$.

Let y_{gi} be the <u>attainment</u> <u>score</u> indicating the degree of attainment of the subject's performance toward the solution of the problem g , which takes on integers 0 through m_{gi} when the subject chooses the strategy i . Figure 4-1 presents several attainment scores assigned to trails which were taken from Example 1. Also Figure 4-2 shows the attainment scores assigned to each edge of the differential strategy tree of the other example, which was presented at the end of Section 3. Hereafter, we shall call this other example Example 2.

A general model for differential strategies concerns the assignment of an operating characteristic to each attainment score y_{gi} of each of the w strategies i for solving the problem g . By such an operating characteristic we mean the conditional probability with which the subject of trait θ chooses the strategy i and obtains the attainment score y_{gi} . We notice, however, that in general, if the subject's performance stopped before branching, there is no way to decide which of the two or more strategies he would have taken. This fact is well represented in the differential strategy tree. For example, in Figure 3-4, $(s_0 \, s_1)$, $(t_0 \, t_1)$, $(u_0 \, u_1)$, $(v_0 \, v_1)$ and $(w_0 \, w_1)$ are a single edge before
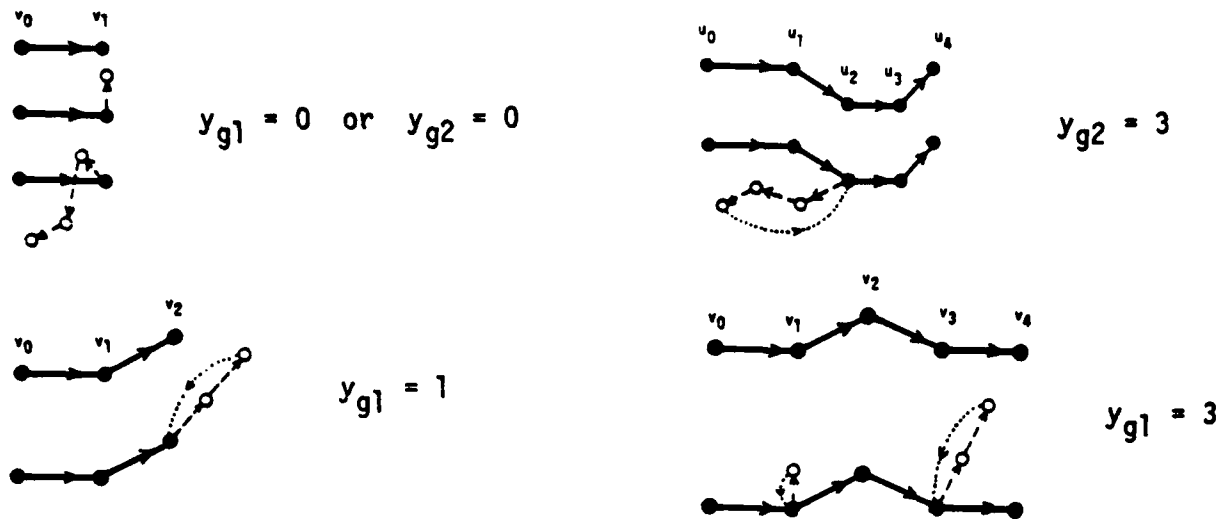
FIGURE 4-1

Examples of Attainment Scores Given to Different
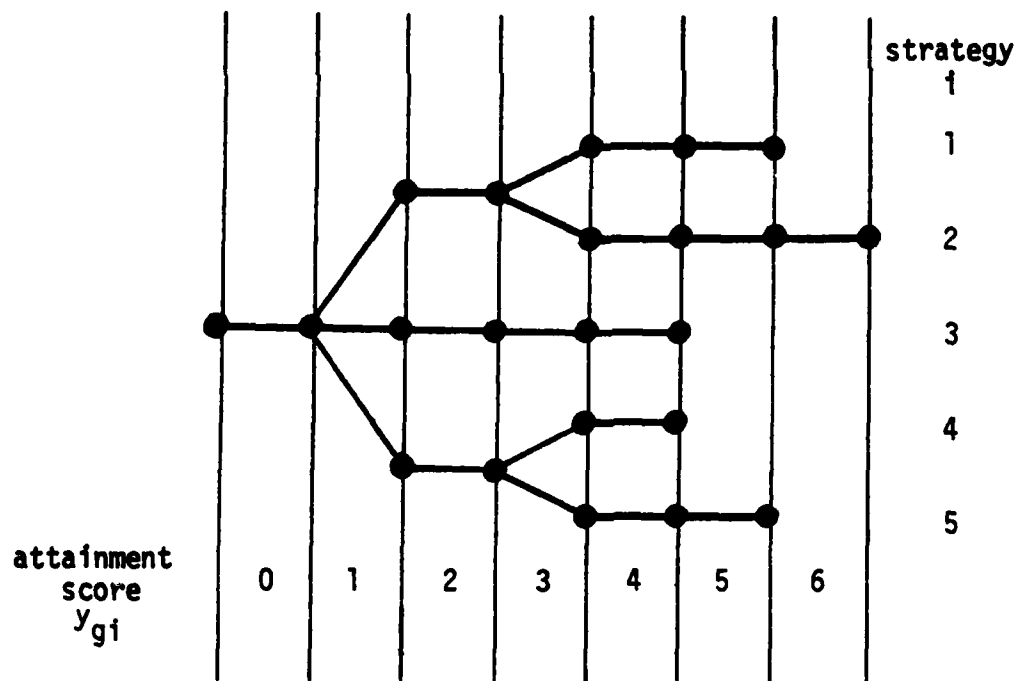Subgraphs in Example 1.

FIGURE 4-2

Attainment Scores Assigned to Each Line of
the Differential Strategy Tree in
Example 2.

the first branching, and as long as the subject's performance stays with this subprocess he could be heading for any one of the five successful strategies. We can also say the same for the edge which is both $(s_1\ s_2)$ and $(t_1\ t_2)$, for the one which is both $(v_1\ v_2)$ and $(w_1\ w_2)$, and so forth. Thus we must assign a single operating characteristic for each edge of the differential strategy tree. Since each edge represents a union of one or more attainment scores, the operating characteristic is to be assigned to each union. For instance, following an appropriate model, a single operating characteristic will be assigned to the union of $y_{gi} = 0$ for $i = 1,2,3,4,5$, and the same model will provide us with an operating characteristic solely for $y_{g4} = 3$, and so on. For convenience, we shall choose the smallest i in each union, and let y^*_{gsi} denote such a union with s for the actual attainment score. In Example 2, for instance, $y^*_{g01} = (y_{g1}=0)\ \text{U}\ (y_{g2}=0)\ \text{U}\ (y_{g3}=0)\ \text{U}\ (y_{g4}=0)\ \text{U}\ (y_{g5}=0)$, and $y^*_{g34} = (y_{g4}=3)$, and none of the four unions, y^*_{g02}, y^*_{g03}, y^*_{g04} and y^*_{g05}, exists.

Let $M_{y^*_{gsi}}(\theta)$ denote the conditional probability with which the subject of trait θ obtains s as his attainment score in one of the strategies which belongs to y^*_{gsi}, with the joint condition that he has already obtained the score $s-1$. Since there is no preceding attainment score for $y_{gi} = 0$, and y^*_{g01} is the union of $y_{gi} = 0$ for all the w strategies, the attainment function $M_{y^*_{g01}}(\theta)$ takes on unity throughout the whole range of θ. On the other hand, since $y_{gi} = m_{gi}+1$ is a hypothesized attainment score which is higher than the full score m_{gi}, the attainment function $M_{y^*_{g(m_{gi}+1)i}}(\theta)$ assumes zero for the entire range of θ for each of the w strategies. Thus we can write

$$(4.1) \qquad M_{y^*_{gsi}}(\theta) \begin{cases} = 1 & i = 1, & s = 0 \\ = 0 & i = 1,2,\ldots,w \ , & s = m_{gi}+1 \ . \end{cases}$$

Note that in (4.1) the first line indicates a single function for the union of $y_{gi} = 0$ for $i = 1,2,\ldots,w$, while the second line indicates w separate functions for $i = 1,2,\ldots,w$.

Hereafter, we shall assume that each attainment function $M_{y^*_{gsi}}(\theta)$ is three-times differentiable with respect to θ . Note that this assumption does not contradict (4.1).

Let $P^*_{y^*_{gsi}}(\theta)$ be the conditional probability assigned to the union of attainment scores y^*_{gsi} , with which the subject of trait θ chooses a strategy which belongs to y^*_{gsi} and obtains the attainment score s or greater. We shall call this function the <u>cumulative</u> <u>operating</u> <u>characteristic</u> <u>of</u> <u>the</u> <u>attainment</u> <u>score</u> <u>union</u> y^*_{gsi} .

From the definitions of this function and the attainment function $M_{y^*_{gsi}}(\theta)$, we can write

$$(4.2) \qquad P^*_{y^*_{gsi}}(\theta) = \prod_{\substack{k=0 \\ *i}}^{s} M_{y^*_{gk(*i)}}(\theta) \ ,$$

where $*i$ indicates the closest integer less than or equal to i for which the union of attainment scores exists. In particular, we have

$$(4.3) \qquad P^*_{y^*_{gsi}}(\theta) \begin{cases} = 1 & s = 0 \\ = M_{y^*_{gsi}}(\theta) & s = 1 \\ = 0 & s = m_{gi}+1 \ . \end{cases}$$

Note that the first line of (4.3) indicates a single function for the

union of $y_{gi} = 0$ for $i = 1,2,\ldots,w$, the second line indicates one or more functions depending upon the branching, and the third line indicates the w separate functions for $i = 1,2,\ldots,w$. In Example 1, the second line includes two functions, while in Example 2 it includes three functions.

Let $A^*_{y^*_{gsi}}(\theta)$ be the first derivative or the natural logarithm of the cumulative operating characteristic $P^*_{y^*_{gsi}}(\theta)$, that is,

$$(4.4) \qquad A^*_{y^*_{gsi}}(\theta) = \frac{\partial}{\partial\theta} \log P^*_{y^*_{gsi}}(\theta) \; .$$

Note that this function is unchanged when $P^*_{y^*_{gsi}}(\theta)$ is multiplied by a constant. To be more specific, let $\Psi(\theta)$ be a function defined by

$$(4.5) \qquad P^*_{y^*_{gsi}}(\theta) = c_1 + (c_2 - c_1)\,\psi(\theta) \; ,$$

where $0 \le c_1 < c_2 \le 1$. The first derivative of the natural logarithm of $P^*_{y^*_{gsi}}(\theta)$ is given by $\frac{\partial}{\partial\theta} \log[c_1 + (c_2-c_1)\psi(\theta)]$, which equals $\frac{\partial}{\partial\theta} \log \psi(\theta)$ if, and only if, $c_1 = 0$. The formula (4.5) has been observed in a somewhat different context (Samejima, ONR/RR-82-1) where $P^*_{y^*_{gsi}}(\theta)$ is replaced by any strictly increasing function of θ with zero and unity as its two asymptotes. We have called the four different types of functions derived from (4.5) Types A, B, C and D, depending upon the values of c_1 and c_2 , i.e., the function is of Type A when $0 = c_1 < c_2 = 1$; of Type B when $0 < c_1 < c_2 = 1$; of Type C when $0 = c_1 < c_2 < 1$; and of Type D when $0 < c_1 < c_2 < 1$, respectively. This implies that the cumulative operating characteristic of Types A and C may share the same function for

$A^*_{y^*_{gsi}}(\theta)$, and we can say the same for those of Types B and D. The necessary and sufficient condition that $M_{y^*_{gsi}}(\theta)$ be strictly increasing in θ is that the inequality

$$(4.6) \qquad A^*_{y^*_{gsi}}(\theta) > A^*_{(y^*_{g(s-1)(*i)})}(\theta) \qquad s = 1,2,\ldots,(m_{gi}+1)$$

holds almost everywhere with respect to θ .

The operating characteristic, $P_{y^*_{gsi}}(\theta)$, defined for the union of the attainment scores Y^*_{gsi} is given by

$$(4.7) \qquad P_{y^*_{gsi}}(\theta) = P^*_{y^*_{gsi}}(\theta) - \sum_{j*} P^*_{y^*_{g(s+1)j}}(\theta) \quad ,$$

where $\sum\limits_{j*}$ indicates the summation over all the strategies j branching from the point which lies immediately after the line representing y^*_{gsi} .

This operating characteristic can be considered as the likelihood function in estimating the subject's latent trait θ . If there are more than one problem satisfying the conditional independence of the attainment scores across the different strategies, given θ , the maximum likelihood estimation of the subject's latent trait can be performed on the basis of the response pattern V , such that

$$(4.8) \qquad V = (y_{11_1}, \; y_{21_2}, \; \cdots, \; y_{gi_g}, \; \cdots, \; y_{ni_n})'$$

for the n problem solving tasks, where i_g is a strategy for solving the problem g and y_{gi_g} is the attainment score when the subject chooses

the strategy i_g for solving the problem g . Let $P_V(\theta)$ be the operating characteristic of the specific response pattern V . We can write

$$(4.9) \qquad P_V(\theta) = \prod_* P_{y^*_{gsi}}(\theta) \quad ,$$

where $\prod_*$ indicates the multiplication over every union y^*_{gsi} to which an element of V belongs.

It is beneficial to search for a family of models which provide us with a unique maximum for every possible response pattern given by (4.8). This can be done as a generalization of the unique maximum condition proposed for the graded response model (cf. Samejima, 1969, 1972). Two of such models will be proposed and observed in Section 5.

The <u>basic function</u>, $A_{y^*_{gsi}}(\theta)$, for the union of attainment scores y^*_{gsi} is defined by

$$(4.10) \qquad A_{y^*_{gsi}}(\theta) = \frac{\partial}{\partial\theta} \log P_{y^*_{gsi}}(\theta) \quad .$$

The maximum likelihood estimate, $\hat{\theta}_V$, of the subject's latent trait based upon his response pattern is given as the solution of the likelihood equation such that

$$(4.11) \qquad \frac{\partial}{\partial\theta} \log P_V(\theta) = \sum_* \frac{\partial}{\partial\theta} \log P_{y^*_{gsi}}(\theta)$$

$$= \sum_* A_{y^*_{gsi}}(\theta) \quad ,$$

where $\sum_*$ indicates the summation over every union y^*_{gsi} to which an

element of V belongs. A sufficient condition that a unique modal point
exists for the likelihood function $P_V(\theta)$ of each and every response
pattern V is that this basic function is strictly decreasing in θ with
non-negative and non-positive values as its two asymptotes, for every
union y^*_{gsi}. This can be shown in the same way that we did for the basic
function $A_{x_g}(\theta)$ of the graded item score x_g (cf. Samejima, 1969).
For brevity, sometimes we call this condition the unique maximum condition.

Multi-correct and multi-incorrect responses have been discussed
briefly (Samejima, 1972, Sections 3.4 and 4.2), and it has been pointed
out that all those response categories can be syndrome categories,
i.e., response categories whose operating characteristics satisfy the
unique maximum condition (Samejima, 1969, 1972). Those cases are included
in Situation B (Samejima, 1972, Section 3.2), where there exist more than
one response category whose operating charateristic is of Type (i) (or of
Type (ii)), and, at least, one response category whose operating characteristic
is of Type (ii) (or of Type (i)), and, in addition there may be one or more
response categories whose operating characteristics are of Type (iii). It
should be recalled that by Types (i), (ii) and (iii) we mean strictly
increasing, strictly decreasing, and unimodal operating characteristics
which satisfy the unique maximum condition. In Situation B, the sum total
of the upper asymptotes of all the operating characteristics of Type (i)
should equal unity. This implies that, as θ tends to positive infinity,
the operating characteristic of each correct answer approaches a finite
positive value less than unity. We notice that such operating characteristics
belong to the Type C model (Samejima, ONR/RR-82-1).

Similarities between the differential strategies in problem solving and the multi-correct responses in testing are obvious. If we consider two or more different strategies which lead to the solution of the problem as two or more different answers to a question, then they will be treated as multi-correct responses. We can see that the concept of multi-correct responses can be transferred to differential strategies, when there exist more than one successful strategy in solving a problem.

V Homogeneous Case

The Homogeneous case of the graded response level has been developed and discussed (Samejima, 1972) as a generalization of a family of models on the dichotomous response level. Sufficient conditions that a model provides us with a unique modal point for the likelihood function of each and every response pattern have been investigated. In the homogeneous case, a sufficient condition is that, for an arbitrary item score x_g ($\neq 0$) , the cumulative operating characteristic $P^*_{x_g}(\theta)$ is of Type A, i.e., strictly increasing in θ with zero and unity as its two asymptotes, and its asymptotic basic function, $\tilde{A}_{x_g}(\theta)$, which is defined by

$$(5.1) \qquad \tilde{A}_{x_g}(\theta) = \frac{\partial}{\partial \theta} [\log \{ \frac{\partial}{\partial \theta} P^*_{x_g}(\theta) \}] \quad ,$$

is strictly decreasing in θ . The satisfaction of this sufficient condition also implies two desirable features of the model such that: 1) the operating characteristic of each graded item score of each item has a single modal point, and 2) those modal points for a single item are

arranged in the same order as the item score itself. The normal ogive and logistic models, which have been generalized from the corresponding models on the dichotomous response level, are two examples of the models which satisfy the above sufficient condition.

These models of the homogeneous case on the graded response level can be generalized to provide us with those which belong to the general model of differential strategies. Let $\Psi(\theta)$ be a function of Type A. We shall consider the cumulative operating characteristic, $P^*_{y^*_{gsi}}(\theta)$, of the union of attainment categories y^*_{gsi} such that

$$(5.2) \qquad P^*_{y^*_{gsi}}(\theta) = \beta_{y^*_{gsi}} \Psi(\theta - \alpha_{y^*_{gsi}}) \; ,$$

where $\alpha_{y^*_{g0i}}$ is negative infinity, $\alpha_{y^*_{g(m+1)i}}$ is positive infinity for $i = 1,2,\ldots,w$, and the values of $\alpha_{y^*_{gsi}}$ are ordered in the same way as those of s, for every strategy, and $\beta_{y^*_{gsi}}$ is a constant which satisfies

$$(5.3) \qquad \sum_{j^*} \beta_{y^*_{gsj}} = \beta_{y^*_{g(s-1)i}} \; ,$$

with $\sum_{j^*}$ indicating the summation over all the strategies j branching from the point of the differentail strategy tree which is located right after the line representing the union $y^*_{g(s-1)i}$. From (5.3) it is obvious that, as far as there is no branching, $\beta_{y^*_{gsi}} = \beta_{y^*_{g(s-1)i}}$.

A sufficient condition that the model satisfies the unique maximum condition is: 1) that the values of the constant $\alpha_{y^*_{gsj}}$ are the same for

all the strategies j which branch from the vertex located immediately after

the edge representing y^*_{gsi} , and 2) that we have

(5.4) $\frac{\partial}{\partial\theta}$ $[\log\{\frac{\partial}{\partial\theta} \Psi(\theta)\}] < 0$

almost everywhere in the domain of θ . To prove this, we obtain from

(4.7), (5.2), (5.3) and the definition of the basic function $A_{y^*_{gsi}}(\theta)$,

which was given by (4.10),

(5.5) $A_{y^*_{gsi}}(\theta) = \frac{\partial}{\partial\theta} \log P_{y^*_{gsi}}(\theta)$

$= \frac{\partial}{\partial\theta} \log[\beta_{y^*_{gsi}} \Psi(\theta-\alpha_{y^*_{gsi}}) - \sum_{j*} \beta_{y^*_{g(s+1)j}} \Psi(\theta-\alpha_{y^*_{g(s+1)j}})]$

where $\sum_{j*}$ indicates the summation over all the strategies j branching

from the vertex which lies immediately after the line representing the

union y^*_{gsi} . By virtue of the first condition, we can rewrite (5.5) in

the form

(5.6) $A_{y^*_{gsi}}(\theta) = \frac{\partial}{\partial\theta} \log [\beta_{y^*_{gsi}} \{\Psi(\theta-\alpha_{y^*_{gsi}}) - (\theta-\alpha_{y^*_{g(s+1)j}})\}]$

$= \frac{\partial}{\partial\theta} \log \{\Psi(\theta-\alpha_{y^*_{gsi}}) - \Psi(\theta-\alpha_{y^*_{g(s+1)j}})\}$.

We notice that, if we replace y^*_{gsi} by the graded item score x_g and

use $\Psi(\theta-\alpha_{x_g})$ as the cumulative operating characteristic $P^*_{x_g}(\theta)$, the

last form of (5.6) is identical with the basic function of the graded item

score, and the left hand side of (5.4) is identical with the corresponding

asymptotic basic function. Thus we can say that all the unions, y^*_{gsi} ,

are equivalent to syndrome response categories (cf. Samejima, 1972, Section 5.2), and a unique maximum is assured for every possible response pattern.

If, for example, $\Psi(\theta)$ is a normal ogive function or a logistic distribution function, then (5.4) is satisfied (Samejima, 1972, Section 5.2), and we can develop the normal ogive model and the logistic model in the context of the general model for differential strategies, and both of them satisfy the unique maximum condition. In these two models, the cumulative operating characteristics are defined by

$$(5.7) \qquad P^*_{y^*_{gsi}}(\theta) = \beta_{y^*_{gsi}}(2\pi)^{-1/2} \int_{-\infty}^{a_g(\theta-b_{y^*_{gsi}})} e^{-u^2/2} \, du$$

and

$$(5.8) \qquad P^*_{y^*_{gsi}}(\theta) = \beta_{y^*_{gsi}}[1 + \exp\{-Da_g(\theta-b_{y^*_{gsi}})\}]^{-1} ,$$

respectively, where a_g (> 0) is the discrimination parameter specific for each problem g , $b_{y^*_{gsi}}$ is a difficulty parameter defined for each union of attainment scores, with $b_{y^*_{g0i}} = -\infty$ and $b_{y^*_{g(m_{gi}+1)i}} = \infty$ and all those values are arranged in the same order as s with respect to each strategy, and D in (5.8) is a scaling factor which assumes 1.7 to retain the same set of parameter values as those in the normal ogive model.

For the purpose of illustration, we shall consider a differential strategy tree shown in Figure 5-1, which was taken from Example 1 given in

Section 1 and has only two successful strategies, or $w = 2$. Note, however, that this tree may represent either one of the digraphs shown in Figure 5-2. Figure 5-3 presents the set of ten cumulative operating characteristics $P_{y_{gsi}^*}^*(\theta)$, with $a_g = 1.00$, $b_{y_{g11}^*} = b_{y_{g12}^*} = -2.50$, $b_{y_{g21}^*} = -1.00$, $b_{y_{g31}^*} = 0.50$, $b_{y_{g22}^*} = -1.80$, $b_{y_{g32}^*} = 0.00$ and $b_{y_{g43}^*} = 2.00$, $\beta_{y_{g11}^*} = 0.60$ and $\beta_{y_{g12}^*} = 0.40$. The cumulative operating characteristic for y_{g01}^* is drawn by a solid line, and the three cumulative operating characteristics for the first strategy are drawn by dotted lines, the four for the second strategy by dashed lines, and the two for y_{g51}^* and y_{g62}^* coincide with the abscissa. Figure 5-4 presents the corresponding operating characteristics, in which all the modal points except for the negative and positive infinities are shown.

VI Single Strategy Case

Suppose that in solving the problem g there is only one successful strategy. Such a single strategy case can be considered as a special case of the multiple strategy case in which $w = 1$. The union of attainment scores, therefore, includes one and only one attainment score, and we can write

$$(6.1) \qquad y_{gsi}^* = (y_g = s) \quad ,$$

where y_g is the attainment score of the unique successful strategy. Thus the three functions, i.e., the attainment function, the cumulative operating characteristic and the operating characteristic, are defined for each attainment score, rather than the union of attainment scores.
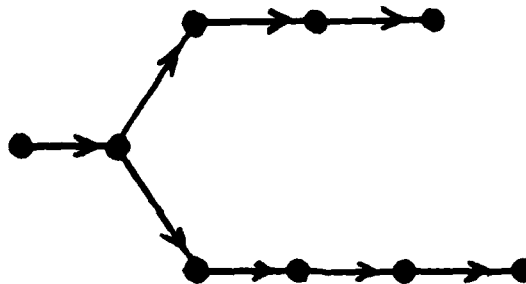
FIGURE 5-1

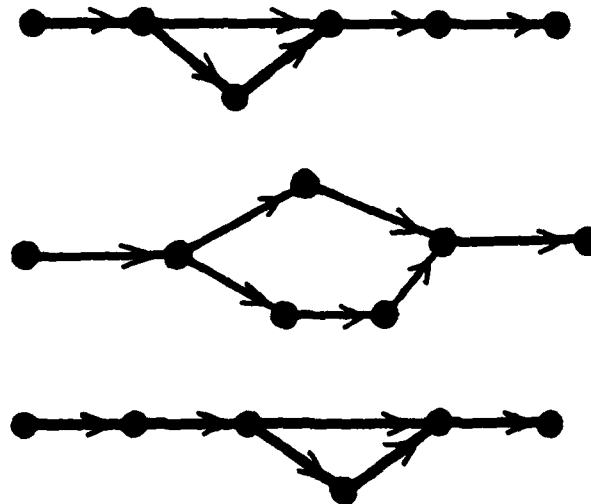Differential Strategy Tree of Two
Strategies Taken from Example 1.

FIGURE 5-2

Three Digraphs Whose Differential Strategy
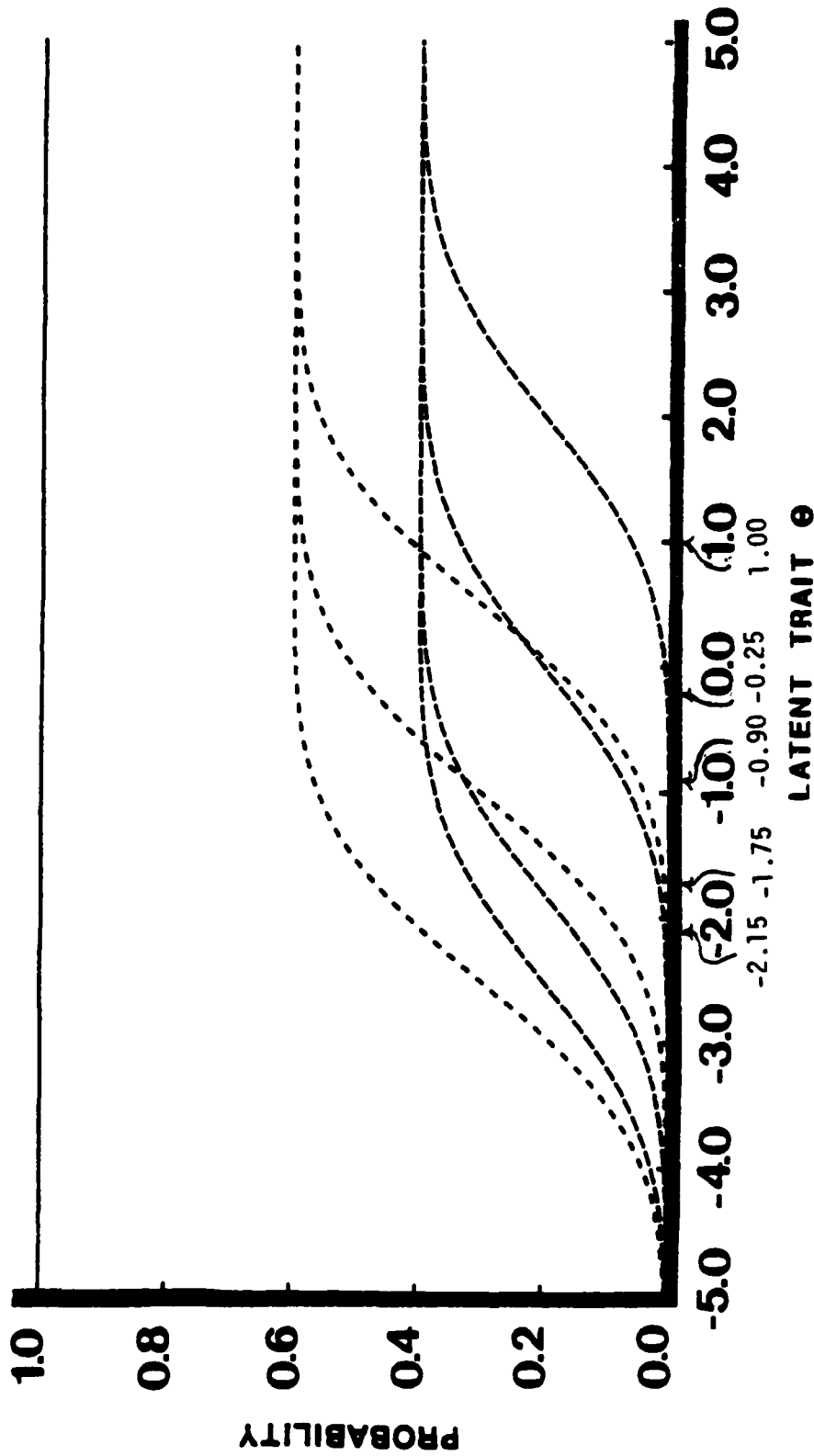Tree Is Given by Figure 5-1.

FIGURE 5-3

Cumulative Operating Characteristic of the Union of ($y_{g1} = 0$) and ($y_{g2} = 0$) (Solid Line),
Those of ($y_{g1} = 1$), ($y_{g1} = 2$) and ($y_{g1} = 3$), Respectively (Dotted Line), and Those of
($y_{g2} = 1$), ($y_{g2} = 2$), ($y_{g2} = 3$) and ($y_{g2} = 4$), Respectively (Dashed Line).

FIGURE 5-4

Operating Characteristic of the Union of $(y_{g1} = 0)$ and $(y_{g2} = 0)$ (Solid Line), Those of $(y_{g1} = 1)$, $(y_{g1} = 2)$ and $(y_{g1} = 3)$, Respectively (Dotted Line), and Those of $(y_{g2} = 1)$, $(y_{g2} = 2)$, $(y_{g2} = 3)$ and $(y_{g2} = 4)$, Respectively (Dashed Line).
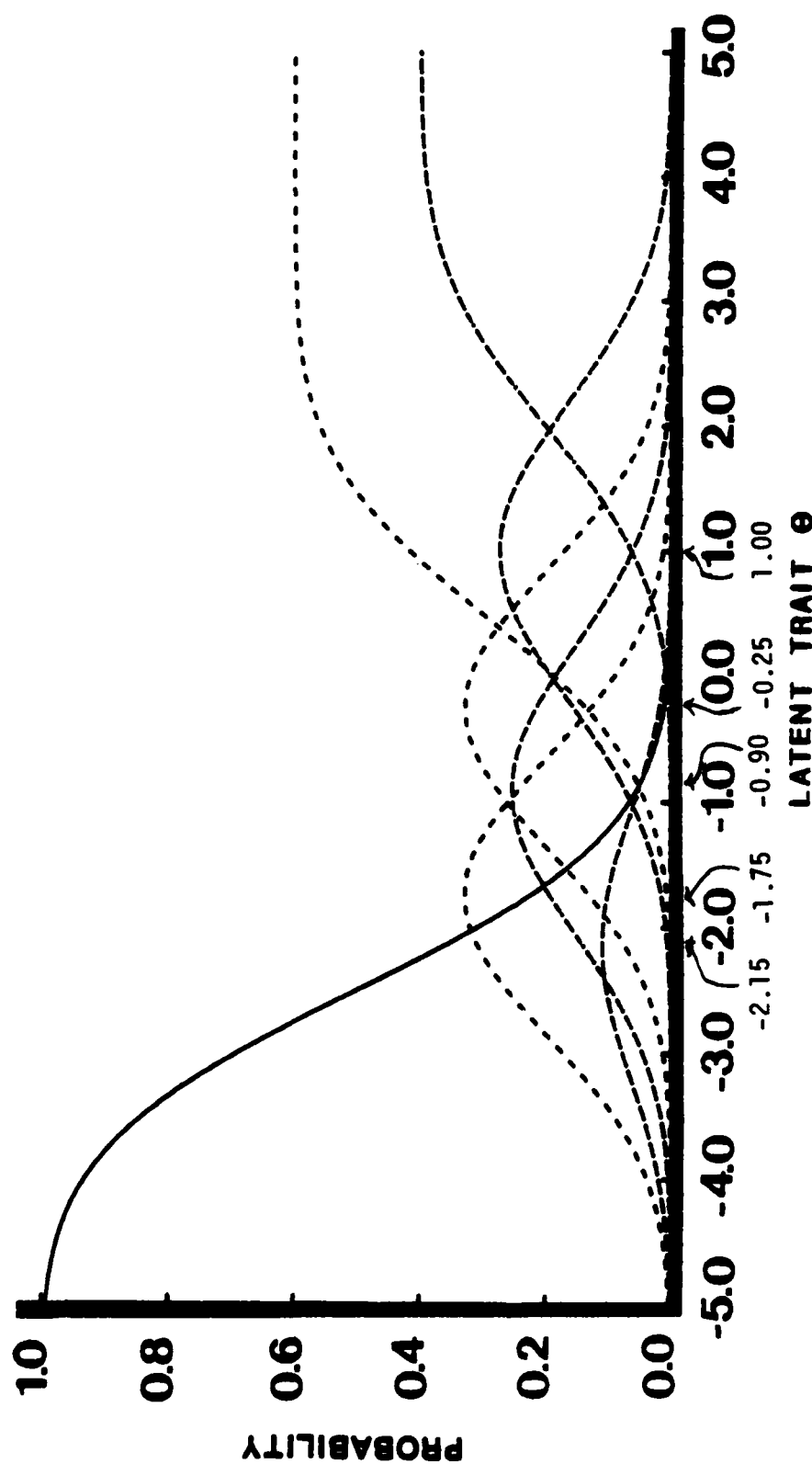
Let $M_{y_g}(\theta)$, $P^*_{y_g}(\theta)$ and $P_{y_g}(\theta)$ denote these three functions for the single strategy case. We can change the formulas (4.1), (4.2), (4.3), (4.7), (4.8) and (4.9) into much simplified forms, such that

$$(6.2) \qquad M_{y_g}(\theta) \begin{cases} = 1 & y_g = 0 \\ = 0 & y_g = m_g+1 \ , \end{cases}$$

$$(6.3) \qquad P^*_{y_g}(\theta) = \prod_{j=0}^{y_g} M_j(\theta) \qquad y_g = 0,1,\ldots,m_g,(m_g+1) \ ,$$

$$(6.4) \qquad P^*_{y_g}(\theta) \begin{cases} = 1 & y_g = 0 \\ = M_{y_g}(\theta) & y_g = 1 \\ = 0 & y_g = m_g+1 \ , \end{cases}$$

$$(6.5) \qquad P_{y_g}(\theta) = P^*_{y_g}(\theta) - P^*_{(y_g+1)}(\theta) \qquad y_g = 0,1,\ldots,m_g \ ,$$

$$(6.6) \qquad V = (y_1, y_2, \ldots, y_g, \ldots, y_n)'$$

and

$$(6.7) \qquad P_V(\theta) = \prod_{y_g \in V} P_{y_g}(\theta) \ ,$$

where m_g indicates the number of the subprocesses involved in solving the problem g .

We notice that these results are identical with those formulas for the graded response model (Samejima, 1972), with the replacement of the item score x_g by the attainment score y_g . Many findings concerning

the graded response model, therefore, can directly be applied for the single strategy case.

Let $A^*_{y_g}(\theta)$ be the first partial derivative of the natural logarithm of $P^*_{y_g}(\theta)$, such that

$$(6.8) \qquad A^*_{y_g}(\theta) = \frac{\partial}{\partial\theta} \log P^*_{y_g}(\theta) \qquad y_g = 0,1,\ldots,(m_g+1) .$$

It has been shown that the necessary and sufficient condition that $M_{y_g}(\theta)$ be strictly increasing in θ is that the inequality

$$(6.9) \qquad A^*_{y_g}(\theta) > A^*_{(y_g-1)}(\theta) \qquad y_g = 1,2,\ldots,(m_g+1)$$

holds almost everywhere with respect to θ (cf. Samejima, 1967, 1972).

In the homogeneous case $P^*_{y_g}(\theta)$ has zero and unity as its two asymptotes for $y_g = 1,2,\ldots,m_g$, and, furthermore, we can write

$$(6.10) \qquad P^*_s(\theta) = P^*_r(\theta - \alpha_{rs}) ,$$

where r and s are two arbitrarily selected attainment categories out of 1 through m_g with $r < s$, and α_{rs} is a positive finite constant. We obtain from (6.8) and (6.10)

$$(6.11) \qquad A^*_s(\theta) = A^*_r(\theta - \alpha_{rs}) .$$

From (6.9) and (6.11) it is obvious that a sufficient, though not necessary, condition that $M_{y_g}(\theta)$ be strictly increasing in θ for

$y_g = 1, 2, \ldots, m_g$ is that $A^*_{y_g}(\theta)$ is strictly decreasing in θ for an arbitrarily chosen attainment category out of 1 through m_g. When m_g tends to positive infinity and α_{rs} for two adjacent real attainment categories tends to zero, this condition tends to the necessary and sufficient condition, for it requires that

$$(6.12) \qquad A^*_{y_g}(\theta) > A^*_{y_g}(\theta + \varepsilon)$$

for any small positive value of θ. Note that this condition is satisfied whenever the unique maximum condition is satisfied. Above all, when the asymptotic basic function, $\tilde{A}^*_{y_g}(\theta)$, which is defined by

$$(6.13) \qquad \tilde{A}^*_{y_g}(\theta) = [\frac{\partial^2}{\partial \theta^2} P^*_{y_g}(\theta)][\frac{\partial}{\partial \theta} P^*_{y_g}(\theta)]^{-1} \quad,$$

is strictly decreasing in θ, not only $M_{y_g}(\theta)$ of each subprocess is strictly increasing in θ, however finely differentiated it may be, but also a unique maximum is assured for the likelihood function of each and every possible response pattern which consists of such attainment scores of different tasks (cf. Samejima, 1972, Sections 5.1 and 5.2). It has been shown (Samejima, 1967, 1972) that in the normal ogive model and in the logistic model on the graded response level (Samejima, 1969), for example, this condition is satisfied. In the former example, we can write

$$(6.14) \qquad P^*_{y_g}(\theta) = (2\pi)^{-1/2} \int_{-\infty}^{a_g(\theta - b_{y_g})} \exp(-u^2/2) \, du$$

and in the latter

$$(6.15) \qquad P^*_{y_g}(\theta) = [1 + \exp\{-Da_g(\theta - b_{y_g})\}]^{-1} \ ,$$

where a_g is the item discrimination parameter and b_{y_g} is the difficulty parameter for each attainment category y_g, and D is a scaling factor which is usually set equal to 1.7 (Birnbaum, 1968). In both models, the upper asymptote of $M_{y_g}(\theta)$ for $y_g = 1, 2, \ldots, m_g$ is unity, while the lower asymptote is zero in the normal ogive model and $\exp[-Da_g(b_{y_g} - b_{(y_g-1)})]$ in the logistic model. This lower asymptote in the logistic model depends upon the distance between the difficulty parameters of the two adjacent attainment categories, assuming zero for $y_g = 1$ and positive numbers less than unity otherwise. In both models, $M_{y_g}(\theta)$ for $y_g = 2, 3, \ldots, m_g$ tends to unity for the entire range of θ as b_{y_g} approaches $b_{(y_g-1)}$, and tends to $P^*_{y_g}(\theta)$ as b_{y_g} departs from $b_{(y_g-1)}$ (cf. Samejima, 1972, Figure 5-2-1).

We shall consider an example of the problem solving task, in which we can assume a single successful strategy. The problem was selected from the thirty paper-and-pencil test items of the LIS Measurement Scale for Non-Verbal Reasoning Ability (Indow and Samejima, 1962, 1966). The test item is called $D_{2.2}$, and is shown in Figure 6-1. There are three Japanese words written by English alphabetical letters, "YAMA," "UMI," and "KAWA," which consist of one, and only one, of the seven one-digit numbers, which are shown in the brackets in Figure 6-1. The sum of the

first two words is supposed to equal the third word, and the examinee is
expected to find out which letter represents which number.

In solving this problem, we notice that: 1) "I" is added to "A"
to produce "A" itself with no carried number added, so "I" must equal
0 ; 2) a similar result is observed for "A" and "U", but 0 is
already assigned to "I", so "U" must be 9 ; 3) "K" must be greater
than "Y" by one, so (Y,K) must be either one of (3,4) , (6,7) and
(7,8) ; 4) since 1 must be carried from the second lowest digit to the
third, two times "M" must be "W" plus ten, so (M,W) must be either
(7,4) or (8,6) ; 5) combining 3) and 4), since no overlappings are
allowed, we we must conclude (Y,K) = (3,4) and (M,W) = (8,6); 6) since
7 is the only remaining number, "A" must be 7 . Note that we
could reverse 1) 2) 3) to 3) 2) 1) , since 3) can be reached
independently and it gives a sufficient information to discover 2) .
Also 4) does not need both 1) and 3) , since we can reach it directly
from 2) . For these reasons, strictly speaking, there are some minor
variations in this strategy, and the above problem is not exactly an
example of the single strategy case. In spite of those variations,
however, our observation tells us that the majority of subjects take the
above strategy, probably because 1) is easier to discover than 3), and
so forth. Table 6-1 presents the attainment scores and the description of
the corresponding subprocess for each score in this example.

It has been reported (Samejima, 1969) that, in their effort of
solving this problem in the paper-and-pencil testing situation, as many
as 187 subjects out of the total of 883 junior high school students

```
    Y A M A           (0 3 4 6 7 8 9)
  +   U M I           A I K M U W Y
  _____         ‖ ‖ ‖ ‖ ‖ ‖ ‖
    K A W A
```

FIGURE 6-1

Item $D_{2.2}$ of LIS Measurement Scale of Non-
Verbal Ability.

TABLE 6-1

Subprocesses of the Task Involved in Solving Item $D_{2.2}$ of
LIS Measurement Scale of Non-Verbal Ability.

Item Score x_g	Attainment Score y_g	Subprocess
0	0	(none)
0	1	I = 0 Left: (3 4 6 7 8 9)
1	2	U = 9 Left: (3 4 6 7 8)
1	3	K = Y + 1 (Y,K) = (3,4) or (6,7) or (7,8)
1	4	W = 2M - 10 (M,W) = (7,4) or (8,6)
1	5	M = 8 W = 6 Y = 3 K = 4 Left: (7)
2	6	A = 7

discovered both $I = 0$ and $U = 9$ and wrote them down on their sheets, although they did not reach the complete solution of the problem. The number of those subjects is almost as large as that of those who reached the complete answer, i.e., 202 . For this reason, using the item score x_g (= 0,1,2) , which is shown in the first column of Table 6-1, the operating characteristic has been estimated for each item score using the normal ogive model on the graded response level. The resultant three operating characteristics, $P_{x_g}(\theta)$ for $x_g = 0,1,2$, have been reported, and are reproduced as Appendix. It has also been reported that this result provides us with a good internal consistency which supports the validity of the model.

VII Discussion and Conclusions

Using digraphs and trees, an attempt was made to give a rationale behind a general model for differential strategies, and some concrete models in the homogeneous case were proposed. The single strategy case was observed as a special case of the multiple strategy case.

A question may arise as to which estimate of the latent trait should be taken if the subject faltered from one strategy to another and did not reach the solution of the problem. One answer to this question may be to take the attainment score of the strategy that he took last, and use its corresponding operating characteristic in estimating his latent trait. An alternative answer may be to compare the resultant estimates of θ obtained by choosing the different strategies the subject has taken and select the highest estimate.

The usefulness of the model is yet to be discovered. The example of the item $D_{2.2}$ of the LIS Measurement Scale provides us, however, with some support for considering cognitive processes in the framework of latent trait theory.

One of the difficulties in any quantification of cognitive data may be their small sample sizes. It will be useful to conduct intensive, observant laboratory studies as a pilot study, and then to use their results in designing more extensive studies, like paper-and-pencil testing, computerized adaptive testing, and so forth, for the purpose of estimating the operating characteristics and eventually the subject's latent trait.

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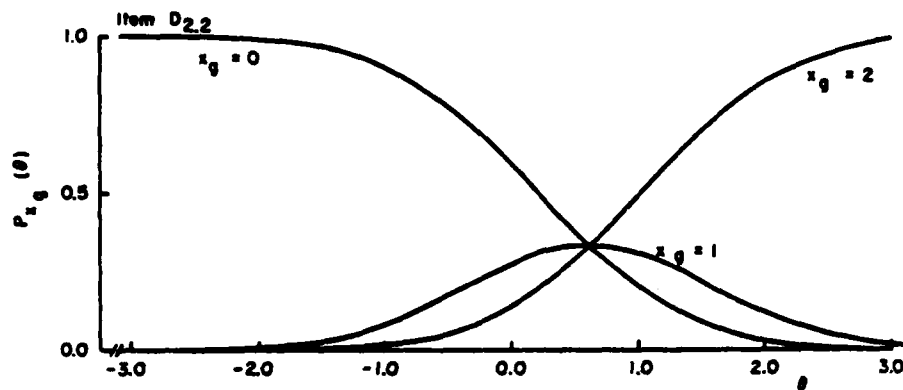   model.   ONR/RR-82-2, 1982.

APPENDIX

FIGURE 5-1

Operating characteristics of graded responses for items C_2 and $D_{2.2}$ of LIS measurement scale for Non-verbal Reasoning Ability on the normal ogive model.

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